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लक्ष्यं विश्वमानम्

WEST BENGAL STATE UNIVERSITY

B.Sc. Honours PART-II Examinations, 2017

MATHEMATICS-HONOURS

PAPER-MTMA-IV

Time Allotted: 4 Hours | Full Marks: 100

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All Symbols are of usual significance.

Group-A

Answer any *two* questions from the following: $10\times 2=20$

1. (a) Prove that the locus of the middle points of the chords of an ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$, which subtend a right angle at the centre of the ellipse is $\left(\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}\right)^2\left(\dfrac{1}{a^2}+\dfrac{1}{b^2}\right)=\dfrac{x^2}{a^4}+\dfrac{y^2}{b^4}$. 5

   (b) The tangents at the extremities of a normal chord of the parabola $y^2=4ax$ meet in a point T. Show that the locus of T is the curve $(x+2a)y^2+4a^3=0$. 5

2. (a) A variable plane has intercept on the co-ordinate axes, the sum of whose square is K^2. Show that the locus of the foot of the perpendicular from the origin to the plane is 5

$$(x^2+y^2+z^2)^2\left(\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}\right)=K^2.$$

(b) Prove that the enveloping cylinders of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$, where generators are parallel to the lines $\frac{x}{0}=\frac{y}{\pm\sqrt{a^2-b^2}}=\frac{z}{c}$ meet the plane $z = 0$ in circles. 5

3. (a) Show that the perpendicular from the origin on the generators of the hyperboloid $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$ lie on the surface 5

$$\frac{a^2(b^2+c^2)^2}{x^2}+\frac{b^2(c^2+a^2)^2}{y^2}=\frac{c^2(a^2-b^2)^2}{z^2}.$$

(b) Reduce the equation $3x^2-y^2-z^2+6yz-6x+6y-2z-2=0$ to its canonical form and find its nature. 5

Group-B

Answer any ***one*** question from the following $10\times1=10$

4. (a) Find the eigen values and eigen functions of the following Boundary Value Problem: $\frac{d}{dx}\left(x\frac{dy}{dx}\right)+\frac{\lambda}{x}y=0,\ y'(1)=0=y'(e^{2\pi}),\ \lambda>0$ 6

(b) Solve: $\frac{dx}{dt}+\frac{dy}{dt}+2y=0$

$\frac{dx}{dt}-3x-2y=0$ 4

5. (a) Apply Lagrange's method to solve: 5

$$z(x+y)\,p+z\,(x-y)\,q=x^2+y^2\,.$$

(b) Find the complete integral of the partial differential equation: 5

$\left(p^2+q^2\right)x=p\,z$ by Charpit's method.

(p and q have their usual meanings)

Group-C

Answer either Question No. 6 or Question No.7 and either Question No. 8 or Question No. 9 13+12=25

6. (a) Prove that the set of all feasible solutions of a linear programming problem is a convex set. 5

(b) Find the dual of the following problem 8

Maximize $z=3x_1+x_2+2x_3-x_4$

Subject to $2x_1-x_2+3x_3+x_4=1$

$x_1+x_2-x_3+x_4=3$

$x_1,\ x_2\geq 0$, x_3, x_4 are unrestricted in sign.

Or

7. (a) Solve the following L.P.P. by Simplex Method 8

Maximize $z=2x_1+3x_2+x_3$

Subject to $-3x_1+2x_2+3x_3=8$

$-3x_1+4x_2+2x_3=7$

$x_1, x_2, x_3\geq 0$

(b) Solve graphically the following rectangular game. 5

		B	
		B_1	B_2
A	A_1	2	7
	A_2	3	5
	A_3	11	2

8. (a) Find the minimum cost of transportation for the following unbalanced transportation problem (symbols have their usual meaning). 7

O_i \ D_i	D_1	D_2	D_3	a_i
O_1	4	3	2	10
O_2	1	5	0	13
O_3	3	8	6	12
b_j	8	5	4	

(b) Prove that if a constant be added to any row and/or any column of the cost matrix of an assignment problem, then the resulting assignment problem has the same optimal solution as the original problem. 5

Or

9. (a) Solve the following assignment problem. 6

		Jobs			
		I	II	III	IV
Machines	A	18	26	17	11
	B	13	28	14	26
	C	38	19	18	15
	D	19	26	24	10

(b) Use dominance to reduce the payoff matrix and solve the game. 6

B

	8	5	8
A	8	6	5
	7	4	5
	6	5	6

Group-D

Answer any ***three*** questions from the following 15×3 = 45

10.(a) Show that the time of descent to a centre of force, the force varying inversely as the square of the distance from the centre, through the first half of its initial distance is to that through its last half as 8

$$(\pi + 2) : (\pi - 2).$$

(b) A particle describes an elliptic orbit under a force which is always directed towards the centre of the ellipse. Find the law of force. 7

11.(a) If t_1 and t_2 be the periods of the vertical oscillations of two different weights suspended by an elastic string and c_1 and c_2 be the statical extensions due to these weights, then prove that $g\left(t_1^2 - t_2^2\right) = 4\pi^2\left(c_1 - c_2\right)$. 7

(b) A particle of mass M is at rest and begins to move under the action of a constant force F in a fixed direction. It encounters the resistance of a stream of fine dust moving in the opposite direction with velocity V, which deposits matter on it at a constant rate μ. Show that its mass will be m when it has travelled a distance $\frac{k}{\mu^2}\left\{m - M\left(1 + \log\frac{m}{M}\right)\right\}$, where $k = F - \mu V$. 8

12.(a) A particle subjected to the central acceleration $\left(\frac{\mu}{r^3}+f\right)$ is projected from an apse at a distance a with a velocity $\frac{1}{a}\sqrt{\mu}$. Prove that at any subsequent time t, $r=a-\frac{1}{2}ft^2$. 8

(b) A particle is describing a parabola about the focus S and moving towards the vertex passes through L, the extremity of the latus rectum, with velocity v. Show that it reaches the vertex after a time $\frac{2\sqrt{2}}{3}\frac{SL}{v}$. 7

13.(a) A particle is projected horizontally with a velocity V along the inside of a rough vertical circle from the lowest point. Prove that if it completes the circle then it will return to the lowest point with a velocity v given by $v^2=V^2e^{-4\pi\mu}-2ag\frac{2\mu^2-1}{1+4\mu^2}\left(1-e^{-4\pi\mu}\right)$, where μ is the coefficient of friction and a is the radius of the circle. 8

(b) Two equal perfectly elastic balls impinge obliquely. If their directions of motion before impact be at right angles, then prove that their directions of motion after impact will also remain at right angles. 7

14.(a) A particle is projected under gravity in a medium whose resistance equals to 'mk' times the velocity. Show that the path of the particle is $y=\frac{g}{k^2}\log\left(1-\frac{kx}{u\cos\alpha}\right)+\frac{x}{u\cos\alpha}\left(u\sin\alpha+\frac{g}{k}\right)$, if it be projected with a velocity u at an angle α to the horizon. 7