B.Sc./Part-II/Hons./MTMA-III/2017

$u = -3$ $\quad v = -1$

$u = -3\omega$ $\quad v = -\omega^2$

$u = -3\omega^2$ $\quad v = -\omega.$

$y = u + v.$

$\omega = -\frac{1}{2}(1+\sqrt{3}i).$

$\omega^2 = -\frac{1}{2}(1-\sqrt{3}i).$

$-3\omega - \omega^2 = \frac{3}{2}(1+\sqrt{3}i) + \frac{1}{2}(1-\sqrt{3}i) = (2+\sqrt{3}i).$

and $-3\omega^2 - \omega = 2-\sqrt{3}i$

Then all valu $= -4, 2\pm\sqrt{3}i$

WEST BENGAL STATE UNIVERSITY

B.Sc. Honours PART-II Examinations, 2017

MATHEMATICS-HONOURS

PAPER-MTMA-III

Time Allotted: 4 Hours Full Marks: 100

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All Symbols are of usual significance.

Group-A

Answer any ***three*** questions from the following $5\times3 = 15$

1. Solve the equation by Cardan's method: 5

$$28x^3 - 9x^2 + 1 = 0.$$

2. If α be an imaginary root of $x^5 - 1 = 0$ then find the equation whose roots are 5

$$\alpha + 2\alpha^4, \alpha^2 + 2\alpha^3, \alpha^4 + 2\alpha.$$

3. Express the equation $x^4 - 2x^3 - 5x^2 + 10x - 3 = 0$ in the form $(x^2 + mx + n)^2 - (px + q)^2 = 0$ and hence solve it. 5

4. Show that $(x+1)^4 + a(x^4+1) = 0$ is a reciprocal equation if $a \neq -1$. Solve the equation when $a = 2$. 5

$\frac{10}{4n} - \frac{1}{2}$

$= \frac{10-4n}{4n}$

5. (a) If a, b, c are all positive real and $abc = k^3$ then prove that 3

$$(1+a)(1+b)(1+c) \geq (1+k)^3 .$$

(b) If a, b, c, be three positive numbers and $a + b + c = 1$ then prove that 2

$$\frac{a}{2-a} + \frac{b}{2-b} + \frac{c}{2-c} \geq \frac{3}{5} .$$

6. (a) If each a, b, c, d be greater than 1 then show that 2

$$8(abcd+1) > (a+1)(b+1)(c+1)(d+1) .$$

(b) If $a_1, a_2, \ldots, a_n$ be positive and $p > q > 0$ then show that 3

$$(a_1^q + a_2^q + \ldots + a_n^q)^p \leq n^{p-q}(a_1^p + a_2^p + \ldots + a_n^p)^q .$$

Group-B

Answer any ***one*** question from the following $10\times1 = 10$

7. (a) Let H be a subgroup of a group G and $a, b \in G$. Show that the left coset aH and bH are identical if and only if $a^{-1} \in H$. 3

(b) Show that every cyclic group is Abelian. Give an example of a group which is Abelian but not cyclic. 3

(c) If $G = (Z, +)$ and $H = (2Z, +)$ then find $[G : H]$, where $[G : H]$ denotes index of H in G. 2

(d) If $a = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$, $b = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$ are two elements of S_3, find a solution of the equation $ax = b$ in S_3. 3

8. (a) Prove that every group of order less than 6 is commutative. 3

(b) Prove that a cyclic group of prime order has no proper non-trivial subgroup. 3

$\frac{5}{4} - \frac{1}{2}\times\frac{11}{6}$ $R_1 - \frac{1}{2}R_2$ $0\ \frac{3}{2} - \frac{2}{3}$

$\frac{5}{4} - \frac{11}{12} = \frac{15-11}{12} = \frac{4}{12} = \frac{1}{3}$ $\frac{11}{4} - \frac{3}{2}\times\frac{11}{6} =$

(c) Show that inverse of the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix}$ is itself. 2

(d) If $f = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 1 & 3 \end{pmatrix}$, $g = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{pmatrix}$ then show that $(fg)^{-1} = g^{-1}f^{-1}$. 2

Group-C

Answer any *two* questions from the following $10\times 2 = 20$

9. (a) Prove that intersection of two subspaces of a vector space is a subspace of the vector space. 2

(b) Let $W = \{(x, y, z) \in \mathbb{R}^3 : x - 4y + 3z = 0\}$. Show that W is a subspace of $\mathbb{R}^3$. Determine a basis of W and hence determine the dimension of W. 4

(c) Extend the set of vectors $\{(-3, 2, -1), (1, -1, -5)\}$ to an orthogonal basis of the Euclidean space $\mathbb{R}^3$ with standard inner product. Find the associated orthonormal basis. 4

10. (a) State replacement theorem. Using this theorem determine a basis of $\mathbb{R}^4$ containing vectors (1, 2, 1, 3), (2, 1, 1, 0) and (3, 2, 1, 1). 1+4

(b) Prove that the eigenvalues of a real symmetric matrix are all real. 5

11. (a) Define Euclidean vector space. Let α, β be two linearly independent vectors in a Euclidean vector space. Prove that $|(\alpha, \beta)| < \|\alpha\|\|\beta\|$. 1+2

(b) Define row rank of a matrix. Use your definition to determine the row rank of the matrix 1+2

$$\begin{pmatrix} 4 & 2 & 5 \\ 3 & 0 & 1 \\ 5 & 4 & 9 \end{pmatrix}.$$

$\frac{21}{4} - \frac{11}{4} = \frac{10}{4}$ $\frac{1}{2} - \frac{1}{2}$ $1\ 0\ 3$ / $0\ 1\ \frac{11}{6}$ / $0\ 0\ 0$

$R_3 - \frac{3}{2}R_2$ $\frac{21}{4} - \frac{3}{2}\times\frac{11}{6}$ $-\frac{11}{4}\times -\frac{2}{3}$ $R_1' = R_1 - \frac{1}{2}R_2$, $\frac{15-11}{12} = \frac{4}{12}$ $\frac{5}{4} - \frac{1}{2}\times\frac{11}{6} = \frac{5}{4} - \frac{11}{12}$ $9 - 3\times\frac{5}{4} = \frac{36-15}{4}$ $4 - \frac{5}{2} = \frac{8-5}{2}$ $1 - \frac{15}{4} = \frac{4-15}{4}$

(c) Correct or justify: 2+2

(i) A set of vectors containing the null vector in a vector space V is linearly independent.

(ii) The union of two subspaces of a vector V is a subspace of V.

12.(a) Prove that a square matrix is orthogonally diagonalisable if and only if it is symmetric. 4

(b) If λ is a non-zero eigenvalue of an orthogonal matrix then show that $\frac{1}{\lambda}$ is also an eigenvalue. 3

(c) Correct or justify: If an $n\times n$ matrix A be non-singular, 0 is not an eigenvalue of A. 3

Group-D

Answer any **two** questions from the following 10×2 = 20

13.(a) Define a subsequence of a sequence of real numbers. Prove that every subsequence of a convergent sequence is convergent. 1+3

(b) If $\{x_n\}$ and $\{y_n\}$ be bounded sequences of real numbers, then prove that 3

$$\underline{\lim}\, x_n + \underline{\lim}\, y_n \le \underline{\lim}\,(x_n + y_n).$$

(c) State Bolzano-Weierstrass theorem on subsequences and illustrate it by taking example. 3

14.(a) State and prove the Intermediate Value Theorem for a continuous function defined on a closed interval. 4

(b) If $\sum_n a_n$ be a convergent series of positive and non-increasing terms, show that $\lim_{n\to\infty} na_n = 0$. 3

(c) Test the convergence of the series 3

$$\frac{1}{2}\cdot\frac{1}{3}+\frac{1\cdot 3}{2\cdot 4}\cdot\frac{1}{5}+\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\cdot\frac{1}{7}+\ldots\ldots\ldots$$

$\frac{2(n+1)+1}{2n+2}$

$\frac{2(n+1)+1}{2n+2-1}$ $\frac{2(n+1)-1}{2n+2-1}$

15.(a) If a real valued function f is continuous on a closed and bounded interval, then prove that it is bounded there. 4

(b) Define uniform continuity. If a real valued function is continuous on a closed and bounded interval then prove that it is uniformly continuous there. 1+3

(c) Examine wheather $f(x)=1-|x|$ has a maximum or minimum at $x = 0$. 2

16.(a) State and prove Rolle's theorem. 5

(b) Obtain Maclaurin's infinite series expansion of $\log(1+x)$, $-1<x\leq 1$. 5

Group-E

Answer any ***five*** questions from the following 5×5 = 25

17. Let $S = \{(a, 0) \in R^2 : a \in R\}$. Show that S is closed but not open. 3+2

18. Define $f(x, y)=\begin{cases} \dfrac{xy}{\sqrt{x^2+y^2}}\ ; & (x, y)\neq(0, 0) \\ 0\ ; & (x, y)=(0, 0) \end{cases}$ 3+2

Show that f is not differentiable at (0, 0) though f is continuous at (0, 0).

19. If $f(x, y)=\begin{cases} xy\ ; & |x|\geq|y| \\ -xy\ ; & |x|<|y| \end{cases}$. 3+2

Show that $f_{xy}(0, 0) \neq f_{yx}(0, 0)$. Which conditions of Schwarz's theorem is not satisfied by f? Justify your answer.

20. State and prove the converse of Euler's theorem on homogenous function of three variables. 1+4

21. Transform the equation $x^2\dfrac{\partial z}{\partial x}+y^2\dfrac{\partial z}{\partial y}=z^2$ by introducing new independent variables $u=x$, $v=\dfrac{1}{y}-\dfrac{1}{x}$ and new function $w=\dfrac{1}{z}+\dfrac{1}{x}$. 5

2n-3 / 2n

2n+3

22. If a function $f(x, y)$ of two variables x and y when expressed in terms of new variables u and v defined by $x = \frac{1}{2}(u+v)$ and $y^2 = uv$ becomes $g(u, v)$, then show that $\frac{\partial^2 g}{\partial u\, \partial v} = \frac{1}{4}\left(\frac{\partial^2 f}{\partial x^2} + \frac{2x}{y}\frac{\partial^2 f}{\partial x\, \partial y} + \frac{\partial^2 f}{\partial y^2} + \frac{1}{y}\frac{\partial f}{\partial y}\right)$. 5

23. Let the double limit $\lim\limits_{(x,y)\to(a,b)} f(x,y)$ exist and equal to A. Let the limit $\lim\limits_{x\to a} f(x, y)$ exist for each fixed value of y in the neighbourhood of 'b' and likewise let the limit $\lim\limits_{y\to b} f(x,y)$ exist for each fixed value of x in the neighboourhood of 'a', then prove that 5

$$\lim_{x\to a}\lim_{y\to b} f(x,y) = A = \lim_{y\to b}\lim_{x\to a} f(x,y).$$

24. Show that the function $u = 3x + 2y - z$, $v = x - 2y + z$ and $w = x(x + 2y - z)$ are dependent and find the relation between them. 2+3

25. Justify the existence and uniqueness of the implicit function $y = y(x)$ for the functional equation $x\cos(xy) = 0$ near the point $\left(1, \frac{\pi}{2}\right)$. Also find $\frac{dy}{dx}\left(1, \frac{\pi}{2}\right)$. 3+2

Group-F

Answer any ***two*** questions from the following 5×2 = 10

26. Find the area of the portion of the circle $x^2 + y^2 = 1$ which lies inside the parabola $y^2 = 1 - x$.

27. Find the coordinate of the centre of gravity of the first arc of the cycloid $x = a(t - \sin t),\ y = a(1 - \cos t)$.

28. Show that the moment of inertia of a thin circular ring of mass M whose outer and inner radii are a and b respectively about an axis through the centre perpendicular to the plane of the ring is $\frac{1}{2}M(a^2 + b^2)$.

29. If the loop of the curve $2ay^2 = x(x-a)^2$ revolves about the line $y = a$, then using Pappus theorem, find the volume of the solid generated.