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B.Sc./Part-I/Hons./MTMA-I/2017

लक्ष्यं विश्वमानम्

WEST BENGAL STATE UNIVERSITY

B.Sc. Honours PART-I Examinations, 2017

MATHEMATICS-HONOURS

PAPER-MTMA-I

Time Allotted: 4 Hours | Full Marks: 100

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Group-A

Answer any *five* questions from the following $5\times5=25$

1. State and prove Fermat's Little theorem. 5

2. (a) Prove that a composite number has at least a prime divisor. 2

   (b) Let a and b be two positive integers such that G. C. D $(a,b)=1$. 3 3

   Prove that G. C. D $(a+b,\ a^2-ab+b^2)=1$ or 3.

3. (a) Expand $\cos^7\theta$ in a series of cosine multiples of θ. 2

   (b) Find the general solution of $\sinh z = 2i$. 3

4. Considering the principal values of logarithms of both sides of the equality $(a+ib)^p = m^{x+iy}$, where $a>b>0,\ p>0,\ m>1,\ x>0,\ y>0$, show that $\tan\left\{\dfrac{y}{x}\log(a^2+b^2)\right\} = \dfrac{2ab}{a^2-b^2}$. 5

5. If $\cosh^{-1}(x+iy)+\cosh^{-1}(x-iy)=\cosh^{-1}a$, where a is a constant > 1, then show that (x, y) lies on an ellipse. 5

6. Find the equation whose roots are the n-th powers of the roots of the equation $x^2-2x+4=0$. Show that the sum of the n-th powers of the roots is $2^{n+1}\cos\frac{n\pi}{3}$. 3+2

7. If α, β, γ be the roots of the equation $x^3+px^2+qx+r=0$, then form the equation whose roots are $\alpha^2+\alpha\beta+\beta^2$, $\beta^2+\beta\gamma+\gamma^2$, $\gamma^2+\gamma\alpha+\alpha^2$. 5

8. Find the equation whose roots are the squares of the roots of the equation $x^4-x^3+2x^2-x+1=0$, use Descartes' rule of signs to deduce that the given equation has no real root. 5

9. Find the condition that the equation $x^n-px^2+r=0$ will have a pair of equal roots. 5

Group-B

Answer any *two* questions from the following $10\times 2=20$

10.(a) Let, A, B, C are three non-empty sets. Prove that 3

$$A\times(B\cap C)=(A\times B)\cap(A\times C).$$

(b) Let A, B and C are three non-empty sets such that 2

$(A\cap C)\cup(B\cap C')=\Phi$ (empty set). Prove that $A\cap B=\Phi$.

(c) Prove that an equivalence relation R on a non-empty set S determines a partitions of S or partitions the set S into equivalence classes. 5

11.(a) A relation ρ defined on the set $\mathbb{N}$ of natural numbers as $a\rho\, b$ iff a divides b. Show that ρ is a partial order relation on $\mathbb{N}$. 2

(b) Prove that the mapping $f: R \rightarrow (-1, 1)$ defined by $f(x) = \dfrac{x}{1+|x|}$ is a bijective mapping. 5

(c) Prove that the inverse of an equivalence relation is an equivalence relation. 3

12.(a) Let $(G, *)$ be a semigroup containing a finite number of elements in which right as well as left cancellation laws hold. Then $(G, *)$ is a group. 5

(b) Define characteristic of an Integral Domain. Prove that the characteristic of an Integral Domain is either 0 or a prime number. 1+4

13.(a) If $a,\ b(\neq 0)$ be two elements of a field $(F,\ +,\ \cdot)$ such that $(ab)^2 = ab^2 + bab - b^2$. Prove that $a = 1$. 5

(b) Prove that the ring $(Z_n, +, \cdot)$ is a field if and only if n is prime. 5

Group-C

Answer any *three* questions from the following 5×3 = 15

14. Prove without expanding 5

$$\begin{vmatrix} 0 & a & b & c \\ a & 0 & c & b \\ b & c & 0 & a \\ c & b & 0 & a \end{vmatrix} = \begin{vmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & c^2 & b^2 \\ 1 & c^2 & 0 & a^2 \\ 1 & b^2 & 0 & a^2 \end{vmatrix}.$$

15. Using Laplace's expansion prove that 5

$$\begin{vmatrix} 0 & a & b & c \\ -a & 0 & d & e \\ -b & -d & 0 & f \\ -c & -e & -f & 0 \end{vmatrix} = (af - be + cd)^2.$$

16. Applying elementary row operations to reduce the following matrix to a row echelon matrix 5

$$\begin{pmatrix} 2 & 0 & 4 & 2 \\ 3 & 2 & 6 & 5 \\ 5 & 2 & 10 & 7 \\ 0 & 3 & 2 & 5 \end{pmatrix}.$$

$$\begin{bmatrix} 1 & 0 & 6 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

17. Solve by Cramer's rule: 5

$$3x + y + z = 3, \quad 2x + 2y + 5z = -1, \quad x - 3y - 4z = 2.$$

18. If A be square matrix and I be an identity matrix of same size as A such that $(I - A)(I + A)^{-1}$ is an orthogonal matrix. Prove that A is a skew-symmetric matrix. 5

19. Reduce the quadratics form $6x^2 + y^2 + 18z^2 - 4yz - 12zx$ to its normal form and examine whether the quadratic form is positive definite or not. 5

Group-D

Answer any *one* question from the following $10 \times 1 = 10$

20.(a) An agricultural farm has 180 tons of Nitrogen fertilizers, 250 tons of phosphate and 220 tons of potash. It is able to sell 3:3:4 mixtures of these substances at a profit of Rs. 15 per ton and 1:2:1 mixtures at a profit of Rs. 12 per ton respectively. Pose a linear programming problem to show how many tons of these two mixtures should be prepared to obtain the maximum profit. 5

(b) Solve the L.P.P. by graphical method: 5

Maximize $Z = 5x_1 + 7x_2$

Subject to $3x_1 + 8x_2 \le 12$,

$x_1 + x_2 \le 2$,

$2x_1 \le 3$,

$x_1, x_2 \ge 0$.

$\frac{3}{2} + x_2 = 2$ $\quad x_2 = \frac{2}{2}$

$x_1 + x_2 = 2$ $\quad 3x_1$ $\quad 3x_1 + 8x_2 = 12$

$2x_1 = 3$ $\quad x_1 = 3/2$ $\quad 3x_1 + 3x_2 = 6$ $\quad x_2 = \frac{6}{5}$ $\quad x_1 = \frac{4}{5}$

21.(a) Show that (2, 1, 3) is a feasible solution of the system of equations: 1+4

$$4x_1 + 2x_2 - 3x_3 = 1,$$

$$-6x_1 - 4x_2 + 5x_3 = -1,$$

$$x_1,\ x_2\ \ x_3 \ge 0.$$

Reduce the feasible solution to a basic feasible solution.

(b) Show that the following system of linear equations has two degenerate basic feasible solutions and the non-degenerate basic solution is not feasible: 3+2

$$3x_1 + x_2 - x_3 = 3,$$

$$2x_1 + x_2 + x_3 = 2.$$

$a_1x + b_1y + c_1z$

$a_2x + b_2y + c_2z$

$a_1b_1 + a_2b_2 + a_3b_3$

Group-E

Section-I

Answer any *three* questions from the following 5×3 = 15

22. Reduce the equation $3x^2 + 2xy + 3y^2 - 16x + 20 = 0$ to its canonical form and state the name of the conic represented by it. 5

23. Prove that the equation to the straight lines through the origin each of which makes an angle α with the straight line $y = x$ is $x^2 - 2xy\sec 2\alpha + y^2 = 0$. 5

24. The straight lines joining the origin to the common points of the curve $ax^2+2hxy+by^2=c$ and the straight line $lx+my=1$ are at right angles. Show that the locus of the foot of the perpendicular from the origin on the straight line is $(a+b)(x^2+y^2)=c$. 5

25. If PSQ and $PS'R$ be two chords of an ellipse through the foci S and S', then prove that $\left(\frac{SP}{SQ}+\frac{S'P}{S'R}\right)$ is independent of the position of P. 5

26. The tangents at two points of the parabola $\frac{l}{r}=1+\cos\theta$ meet at T. 5

Show that $SP \cdot SQ = ST^2$, where S is the focus.

Section-II

Answer any *three* questions from the following. $5\times3=15$

27. The co-ordinates of the points A, B, C, D are $(1, 1, 1)$, $(-1, 3, -3)$, $(3, -1, 2)$ and $(-3, 5, -4)$ respectively. Show that the lines AB and CD intersect and find the point of intersection. 5

28. Prove that the acute angle between the lines whose direction cosines (l, m, n) are given by the relations $l+m+n=0$ and $l^2+m^2-n^2=0$ is $\frac{\pi}{3}$. 5

29. A variable plane makes intercepts on the co-ordinate axes, the sum of whose squares is constant and equal to k^2. Show that the locus of the foot of the perpendicular from the origin to the plane is 5

$$\left(\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}\right)(x^2+y^2+z^2)=k^2.$$

30. Find the equation of the projection of the line $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-4}{4}$ on the plane $x+3y+z+5=0$. 5

31. Find the equation of the plane containing the line $\frac{y}{b}+\frac{z}{c}=1$, $x=0$ and parallel to the line $\frac{x}{a}-\frac{z}{c}=1$, $y=0$. If $2d$ is the shortest distance between the given lines, prove that 5

$$\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{d^2}.$$

$(af-be+cd)$

$-2abef$

$+2acdf -2bced$

$hm+mn+ln$

$n(-l-n)+mn+n(-m-n)$

$-nl-hn+mn-mn$