West Bengal State University

B.A./B.Sc./B.Com (Honours, Major, General) Examinations, 2014

PART – III

MATHEMATICS — HONOURS

Paper – VIII (A)

Duration : 2 Hours] [Full Marks : 50

The figures in the margin indicate full marks.
(Notations used have their usual meanings.)

Group – A

Section – I

(Linear Algebra)

Answer any *one* question from the following. $1 \times 10 = 10$

a) Let V and W be two vector spaces over a field F. When a linear mapping $T: V \rightarrow W$ is defined to be invertible ? 1

b) Let V and W be two vector spaces over a field F. Prove that a necessary and sufficient condition for a linear mapping $T: V \rightarrow W$ to be invertible is that T is one-to-one and onto. 4

c) A linear mapping $T: R^3 \rightarrow R^3$ is defined by

$$T(x_1, x_2, x_3) = (2x_1 + x_2 - x_3, x_2 + 4x_3, x_1 - x_2 + 3x_3), (x_1, x_2, x_3) \in R^3.$$

Find the matrix representation of T relative to the ordered basis $(0, 1, 1), (1, 0, 1), (1, 1, 0)$ of R^3. 5

a) If V and W are two finite dimensional vector spaces and $T: V \rightarrow W$ is a linear transformation then show that dim V = nullity of T + rank of T. 4

b) Find the linear transformation $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$, if

$T(1, 0, 0) = (2, 3, 4)$

$T(0, 1, 0) = (1, 5, 6)$

and $T(1, 1, 1) = (7, 8, 4)$

Also find its matrix representation with respect to $\{ (1, 0, 0), (0, 1, 0), (1, 1, 1) \}$. Is T invertible ? Justify. 2 + 2 + 2

Section – II

(Modern Algebra)

Answer any *one* question from the following. $1 \times 8 = 8$

3. a) Let H be a subgroup of a group G. Then prove that H is normal in G iff $xhx^{-1} \in H$ for all $x \in G$ and for all $h \in H$. 3

b) If every cyclic subgroup of a group G be normal in G then prove that every subgroup of G is normal in G. 2

c) Let (G, o) and (G', $*$) be two groups and let $\phi : G \to G'$ be a homomorphism. Then prove that

i) $\phi(e_G) = e_{G'}$

ii) $\phi(a^{-1}) = [\phi(a)]^{-1} \; \forall a \in G$

Let $(Q,+)$ be the group of rational numbers under addition and $(Q^+,\bullet)$ be the group of positive rational numbers under multiplication. Then show that $(Q,+)$ and $(Q^+,\bullet)$ are not isomorphic. 1 + 1 + 1

4. a) Let H be a subgroup of a commutative group G. Then prove that the quotient group G/H is commutative. Is the converse true ? Justify. 3

b) Let G be a group and H be a normal subgroup of G. Then prove that there exists an epimorphism $f : G \to G/H$ such that $\ker f = H$. 3

c) Show that any infinite cyclic group (G, $*$) is isomorphic to the group ($\mathbb{Z}$, +). 2

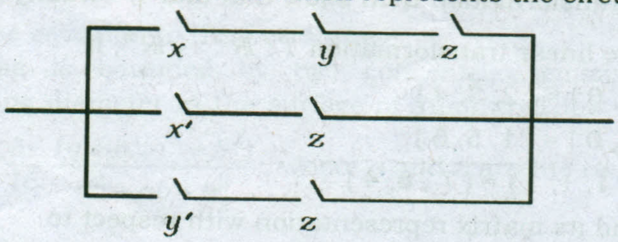
Section – III

(Boolean Algebra)

Answer any *one* question from the following. $1 \times 7 = 7$

5. a) Define a Boolean algebra. Prove that $P(A)$, the power set of a non-empty set A, forms a Boolean algebra with respect to the set union, intersection and complementation. 1 + 3

b) Find the Boolean function which represents the circuit



Find a simpler equivalent switching circuit, if any. 3

6 a) Prove that the set S of all positive divisors of 70 forms a Boolean algebra $(S, \vee, \wedge, ')$, where

$a \vee b$ = l.c.m. of a, b

$a \wedge b$ = g.c.d. of a, b

$a' = \frac{70}{a}$. 4

b) Draw the circuit which realises the function f given in the table : 3

x	y	z	f
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

Group – B

(Differential Equations-III)

Answer any *one* question from the following. 1 × 15 = 15

7. a) Solve the equation $\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + 3y = 0$ in series near the ordinary point $x = 0$. 5

b) Using change of scale property, evaluate $L\{3\cos 6t - 5\sin 6t\}$ and hence show that $L\{e^{-2t}(3\cos 6t - 5\sin 6t)\} = \frac{3(s-8)}{s^2+4s+40}$. 5

c) Solve using Laplace transform, $y'' + 2y' + 2y = t$ given that $y(0) = y'(0) = 1$. 5

8. a) Obtain series solution of $\frac{d^2y}{dx^2} + y = 0$ near the ordinary point $x = 0$. 5

b) If $F(s) = \frac{1}{(s^2+a^2)(s^2+b^2)}$ $(a \neq b)$, then find $f(t)$, where $f(t) = L^{-1}\{F(s)\}$. 5

c) Find the solution of the initial value problem $(D^2 - D - 2)y = 20\sin 2t$, given that $y = -1$, $Dy = 2$ at $t = 0$, where $D \equiv \frac{d}{dt}$. 5

Group – C

(Tensor Calculus)

Answer any *one* question from the following. $1 \times 10 = 10$

9. a) If f is an invariant, determine whether $\frac{\partial^2 f}{\partial x^p \partial x^q}$ is a tensor. 3

b) If $A_{ij}^k B_k^{jl} = 0$ for every B_k^{jl}, prove that A_{ij}^k vanishes identically. 4

c) Show that in an n-dimensional space a covariant skew-symmetric tensor of second order has at most $\frac{1}{2}n(n-1)$ different arithmetic components. 3

10. a) Show that in a Riemannian space V_n of dimension n with metric tensor g_{ij}, $\left\{ \begin{matrix} i \\ ij \end{matrix} \right\} = \frac{\partial}{\partial x^j}(\log \sqrt{g})$. 4

b) If A_{ij} is a symmetric tensor then show that $A_{ij,k}$ is symmetric in i and j. 2

c) Line element of two neighbouring points $P(x^i)$ and $Q(x^i + dx^i)$ in a 3-dimensional space is given by $ds^2 = (dx^1)^2 + 2(dx^2)^2 + 3(dx^3)^2 - 2dx^1dx^2 + 4dx^2dx^3$. By this line element, does the above space form a Riemannian space ? Justify it. 4