West Bengal State University

B.A./B.Sc./B.Com. (Honours, Major, General) Examinations, 2012

PART-III

MATHEMATICS - (HONOURS)

Paper– VIII-A

Duration : 2 Hours Full Marks :

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

GROUP – A

SECTION - I

(Linear Algebra)

Answer any *one* question :

1. a) Let V and W be vector spaces over a field F. If $\{\alpha_1, \alpha_2, \ldots, \alpha_n\}$ is a basis of V and $\beta_1, \beta_2, \ldots, \beta_n$ are arbitrary elements in W, then show that there exists one and only one linear mapping $T : V \to W$ such that $T(\alpha_i) = \beta_i$, $i = 1, \ldots n$.

   b) Show that $T : R^3 \to R^3$ defined by $T(x, y, z) = (x - y, x + 2y, y + 3z)$, $(x, y, z) \in R^3$ is a linear map. Also prove that T is invertible and find T^{-1}.

   2 + 1 +

2. a) If V and W are finite dimensional vector spaces over a field F then show that V and W are isomorphic if and only if $\dim V = \dim W$.

   b) Find the linear map $T : R^3 \to R^3$, if

   $T(0,1,1) = (1,0,1)$

   $T(1,0,1) = (2,3,4)$

   $T(1,1,0) = (1,2,3)$

   Determine the matrix of T with respect to the standard basis of R^3. 2 +

   c) If $T : V \to W$ is an invertible linear map then show that $T^{-1} : W \to V$ is also a linear map.

SECTION-II

(Modern Algebra)

Answer any one question :

3. a) If $\phi : G \to G'$ is an isomorphism, where G and G' are finite groups then prove that $O(a) = O(\phi(a))$ for every $a \in G$. Use the result to show that the groups $(Z_4, +)$ and Klein's four group V are not isomorphic. 2 + 2

   b) If (H, o) is a normal subgroup of a group (G, o), then prove that the quotient group $(G/H, *)$ is Abelian if and only if $x \, o \, y \, o \, x^{-1} \, o \, y^{-1} \in H, \ \forall x, y \in G$. 4

4. a) If H is a subgroup of a group G and $[G : H] = 2$, then prove that H is normal in G. 2

   b) If ϕ is a homomorphism from a group (G, o) to a group $(G', *)$ then prove that ϕ is one-to-one if and only if $\ker\phi = \{e_G\}$, where e_G is the identity element in G.

   K is a multiplicative commutative group of order 8. Prove that $\phi : K \to K$ defined by $\phi(x) = x^3$, $x \in G$ is an isomorphism. 3 + 3

SECTION – III

(Boolean Algebra)

Answer any *one* question :

5. a) In a Boolean algebra B, show that,

   $(a+b)(b+c)(c+a) = ab + bc + ca$, for all $a, b, c \in B$. 3

   b) Express the Boolean expression

   $((x'+y)' + (x'+y))'$ in CNF in the variables present in the expression. 4

6. a) Change the following function from CNF to the DNF :

   $(a+b+c)(a'+b'+c)(a+b'+c')(a'+b+c')$. 3

   b) We wish a light in a room to be controlled independently by three wall switches located at the three entrances of the room in such a way that flicking any one of them will change the state of the light (on to off and off to on). Design a simple series-parallel switching circuit which will do the required job. 4

GROUP-B

(Differential Equations-III)

Answer any *one* question :

7. a) Obtain the series solution of $\frac{d^2y}{dx^2} - 4y = 0$ satisfying $y(0) = 1$ and y'

b) Find the inverse Laplace transform of $\frac{5s+3}{(s-1)(s^2+2s+5)}$.

c) Using Laplace transform solve

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{-x}, \text{ when } y(0) = y'(0) = 0.$$

8. a) Obtain series solution of $\frac{d^2y}{dx^2} + y = 0$ near the ordinary point $x = 0$.

b) Find the inverse Laplace transform of $\frac{s+4}{s(s-1)(s^2+4)}$.

c) Using the method of Laplace transform solve

$$\frac{d^2x}{dt^2} + \frac{2dx}{dt} + x = 3te^{-t}, \text{ when } x(0) = 4, \ x'(0) = 2.$$

GROUP-C

(Tensor Calculus)

Answer any one question.

9. a) Show that the components of a tensor of type (0, 2) can be express sum of a symmetric tensor and a skew symmetric tensor of the same

b) If a_{ij} is a skew symmetric tensor then prove that

$$(\delta_i^l \delta_l^k + \delta_l^l \delta_j^k) a_{ik} = 0.$$

c) Show that the only non-vanishing Christoffel symbols of the second kind for V^2 with line element $ds^2 = (dx^1)^2 + \sin^2 x^1 (dx^2)^2$ are

$$\left\{ \begin{matrix} 1 \\ 22 \end{matrix} \right\} = -\sin x^1 \quad and \quad \left\{ \begin{matrix} 2 \\ 12 \end{matrix} \right\} = \left\{ \begin{matrix} 2 \\ 21 \end{matrix} \right\} = \cot x^1 .$$

3 + 2 + 5

10. a) Show that $[ij, k] + [kj, i] = \dfrac{\partial g_{ik}}{\partial x^j}$. Use it to show that for any symmetric tensor a^{ij},

$$a^{jk} [ij, k] = \frac{1}{2} a^{jk} \frac{\partial g_{jk}}{\partial x^i}.$$

b) If A^i and B^i are two non-null vectors such that $g_{ij} U^i U^j = g_{ij} V^i V^j$ where $U^i = A^i + B^i$ and $V^i = A^i - B^i$, then show that A^i and B^i are orthogonal.

c) Show that $g_{ik,j} = 0$ and $\delta^i_{k,j} = 0$.

(2 + 3) + 2 + (2 + 1)
