West Bengal State University

B.A./B.Sc./B.Com. (Honours, Major, General) Examinations, 2012

PART–III

MATHEMATICS — Honours

Paper–VI

Duration : 4 Hours Full Marks : 100

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

GROUP - A

(Marks : 50)

(Probability and Statistics)

Answer any *two* questions from Q. Nos. 1 to 3 and any *one* from Q. Nos. 4 and 5.

1. Answer any *three* of the following questions : $3 \times 5 = 15$

a) What is meant by the term 'statistical regularity' ? Explain how the frequency definition of the probability of a random event related to the concept of statistical regularity.

Starting from the frequency definition of probability establish the following :

$P(A_1 + A_2 + \ldots\ldots + A_n) = P(A_1) + P(A_2) + \ldots\ldots + P(A_n)$ where A_is are mutually exclusive random events for $i = 1, 2, \ldots\ldots n$. $1 + 1 + 3$

b) State and prove Bayes' theorem. $1 + 4$

c) If the events A and B are independent then prove that the events $\overline{A}$ and B are independent. Consider events A and B such that $P(A) = \frac{1}{4}$, $P\left(\frac{B}{A}\right) = \frac{1}{2}$, $P\left(\frac{A}{B}\right) = \frac{1}{4}$.

Find $P\left(\frac{\overline{A}}{\overline{B}}\right)$ and $P\left(\frac{A}{\overline{B}}\right)$. $3 + 2$

d) Four students have identical umbrellas which they keep in some definite [cut off] while attending class. After the class each student select an umbrell[cut off] random and goes home. What is the probability that at least one umbrella [cut off] to its original owner ?

e) If $\{A_n\}$ be a monotone sequence of random events then prove [cut off] $P\left(\lim_{n \to \infty} A_n\right) = \lim_{n \to \infty} P(A_n)$.

2. Answer any *three* questions of the following : 3 × 5

a) For any three random events A, B and C establish the following :

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C).$$

b) If $F(x)$ be the distribution function of a random variable X, then prove that

i) $F(a) \leq F(b)$ if $a < b$

ii) $F(x) \to F(a)$ as $x \to a + 0$

iii) $F(a) - \lim_{x \to a - 0} F(x) = P(X = a)$. 1 + 2

c) In the equation $x^2 + 2x - q = 0$, q is a random variable uniformly distributed o[cut off] the interval $(0, 2)$. Find the distribution function of the larger root.

d) Define Poisson distribution. Prove that the sum of two independent Poiss[cut off] variates having parameters μ_1 and μ_2 is a Poisson variate having parame[cut off] $\mu_1 + \mu_2$. 2 [cut off]

e) If the probability density function of a random variable X is given [cut off] $f(x) = \frac{e^2}{\sqrt{\pi}} e^{-(x^2+2x+3)}$, $-\infty < x < \infty$, find the expectation and variance of t[cut off] distribution. 2 +

3. Answer any *three* of the following questions : $3 \times 5 = 15$

a) Prove that under certain conditions stated by you, the binomial distribution tends to Poisson distribution in the limits. 1 + 4

b) If X be a normal (m, σ) variate then prove that, $\mu_{2r+2} = \sigma^2 \mu_{2r} + \sigma^3 \frac{d\mu_{2r}}{d\sigma}$. Hence find the coefficient of Kurtosis β of this distribution. 3 + 2

c) State and prove Tchebycheff's inequality. 1 + 4

d) If $\{X_i\}$ be a sequence of independent random variable such that for each i, $E(X_i) = m_i$, $Var\, X_i = \sigma_i^2 < \sigma^2 < \infty$, use Tchebycheff's inequality to show that $\sum_{i=1}^{n} \frac{X_i}{n} = \sum_{i=1}^{n} \frac{m_i}{n} \xrightarrow[\text{in } p]{} 0$ as $n \to \infty$. 5

e) State Bernoulli's theorem.

Also state the law of large numbers. Obtain Bernoulli's theorem as a particular case of the law of large numbers for equal components. 1 + 1 + 3

4. a) Explain what are meant by a statistic and its sampling distribution.

Let $x_1, x_2, \ldots, x_n$ be a random sample of size n (> 1) from a normal (m, σ) population. Find the sampling distribution of the statistic $t = \frac{(\bar{x} - m)\sqrt{n}}{s}$, where $\bar{x}$ is the sample mean and $(n-1)s^2 = \sum_{i=1}^{n} (x_i - \bar{x})^2$. 1 + 1 + 7

b) A random sample $(x_1, x_2, \ldots, x_n)$ of size n is taken from a population with variance σ^2. If $S^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}$, $\bar{x} = \frac{1}{n} \sum_{i=i}^{n} x_i$, show that the variance of the sampling distribution of S^2 is given by $\frac{1}{n}\left\{\mu_4 - \frac{n-3}{n-1}\sigma^4\right\}$, where μ_4 is the fourth order central moment of the population. 7

c) In a random sample of 400 articles 40 are found to be defective. Obtain 95 confidence interval for the true proportion of defectives in the population such articles. Given $\int_0^{1\cdot96} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx = 0\cdot4750$.

5. a) Explain the following terms with examples :

i) Null hypothesis,

ii) Alternative hypothesis.

Explain the concept of a critical region and power of a test. 1 + 1 + 1 +

b) For the normal (m, σ) population, where m is known, test the null hypothesis $H_0 : \sigma = \sigma_0$ against an alternative

$H_1 : \sigma = \sigma_1$ on the basis of a sample $\left(x_1, x_2, ... x_n\right)$ from the population.

c) Find the maximum likelihood estimator of the parameters of a (n, σ population on the basis of a random sample of size n drawn from th population. Examine whether the estimators are unbiased and consistent.

GROUP - B

(Marks : 50)

(Numerical Analysis and Computer Programming)

Answer any *three* questions from Section - I and any *two* from Section - II.

SECTION - I

(Marks : 30)

6. a) What are the different sources of computational errors in a numerical computational work ? Two lengths X and Y are measured approximately up to 3s as $X = 3\cdot32$ cm and $Y = 5\cdot39$ cm. Estimate the error in the computed value of $X + Y$. 5

b) Define the kth order difference of a function $f(x)$. Prove also that for equally spaced interpolating points $x_i = x_0 + ih,\ h > 0,\quad i = 0, 1, \ldots, n$

$$\Delta^k_{y_0} = \sum_{i=0}^{k} (-1)^i \binom{k}{i} y_{k-i} \text{ where } y_i = f(x_i).$$ 5

7. a) Define divided difference of two arguments x_0, x_1 and prove that

$$f(x_0, x_1, \ldots, x_n) = \sum_{i=0}^{n} \frac{f(x_i)}{(x_i - x_0)(x_i - x_1)\ldots(x_i - x_{i-1})(x_i - x_{i+1})\ldots(x_i - x_n)}.$$ 1 + 4

b) Obtain Lagrange's interpolation formula (without the error term). 5

8. a) Explain the Regua-Falsi method in finding a real root of the equation $f(x) = 0$. State where it is different from secant method. 5

b) Describe Gauss' elimination method for numerical solution of a system of n linear equations with n variables. Give estimates of the 'count' number of this method for large n. 5

9. a) Describe Gauss' elimination method to solve numerically a system of n linear equations with n unknown variables. What are the demerits of this method and how can it be avoided ? 6

b) Describe Bisection method for computing a simple real root of $f(x) = 0$. Give a geometrical interpretation of the method and also the error estimate. 4

10. Explain briefly Euler's method to solve the differential equation $\frac{dy}{dx} = f(x, y),\ y = y_0$ when $x = x_0$. How can the method be modified ? 6 + 4

SECTION - II

(Marks : 20)

11. Answer any *four* of the following : $4 \times 2\frac{1}{2} =$

i) What is memory ? Write the name of three types of memory that are used modern computers.

ii) What are the functions of ALU ?

iii) Write a short note on software.

iv) Draw a flow chart to find the HCF of two distinct positive integers m and n.

v) Use 2's complement to compute $(110110)_2 - (100100)_2$.

vi) Find the CNF of $xy' + x'y$.

vii) Draw a flowchart to obtain the LCM of two positive integers.

12. a) Write a FORTRAN 77 - program to arrange the following set of real numbers ascending order :

10·1, 3·2, 7·5, 13·9, 9·4, 14·1, 8·5, 1·9.

b) Given the values of a, b, c, the lengths of three line segments. Write FORTRAN program to test whether they can form a triangle or not.

13. a) Explain the uses of 'switch' and 'break' statements in *C* with suitable examples. 5

b) Write a *C*-program to compute the following sum $S = 1^4 + 3^4 + 5^4 + ...$ until *S* remains less than 10,000. 5
