WEST BENGAL STATE UNIVERSITY

B.Sc. Honours PART-III Examination, 2016

MATHEMATICS-HONOURS

MTMA-VIII

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks. Candidates should answer in their own words and adhere to the word limit as practicable. Symbols are of usual significance.

Group- A

(Full Marks- 25)

Section-I

(Linear Algebra)

Answer *one* question from the following: $10\times 1 = 10$

1. (a) Prove that two finite dimensional vector spaces V and W over a field F are isomorphic if and only if $\dim V = \dim W$. 4

   (b) The matrix of a linear mapping $T : \mathbb{R}^3 \to \mathbb{R}^2$ relative to the ordered bases (0, 1, 1), (1, 0, 1), (1, 1, 0) of $\mathbb{R}^3$ and (1, 0), (1, 1) of $\mathbb{R}^2$ is 6

$$\begin{pmatrix} 2 & 1 & 4 \\ 1 & 2 & 0 \end{pmatrix}.$$

   Find the explicit representation of T and the matrix of T relative to the ordered bases (1, 0, 1), (1, 1, 0), (0, 1, 1) of $\mathbb{R}^3$ and (1, 2), (1, 1) of $\mathbb{R}^2$.

2. (a) Let V and W be two linear spaces over a field F and $B = \{\alpha_1, \alpha_2, \ldots\ldots, \alpha_n\}$ be a basis for V. Prove that for any n elements β_i $(i = 1, 2, .., n)$ of W, there exists a unique linear transformation $T : V \rightarrow W$ such that $T(\alpha_i) = \beta_i$ $(i = 1, 2, \ldots, n)$. 4

   (b) Let $T : \mathbb{R}^4 \rightarrow \mathbb{R}^2$ be defined by $T(x_1, x_2, x_3, x_4) = (3x_1 - 2x_2 - x_3 - x_4, x_1 + x_2 - 2x_3 - 3x_4)$, for any $(x_1, x_2, x_3, x_4) \in \mathbb{R}^4$. Prove that T is a linear transformation. Hence find rank T, nullity T and a basis of ker T. 6

Section- II
(Modern Algebra)

Answer any *one* question from the following: 8×1= 8

3. (a) Let $f : G \rightarrow G'$ be a homomorphism of groups. Prove that ker f is a normal subgroup of G. Also prove that f is one-to-one if and only if ker $f = \{e_G\}$, where e_G represents the identity element of G. 4

   (b) Let G be a group and H be a subgroup of G such that $aba^{-1}b^{-1} \in H$ for all $a, b \in G$. Prove that H is a normal subgroup of G and the quotient group $\dfrac{G}{H}$ is commutative. 4

4. (a) Prove that an infinite cyclic group is isomorphic to the additive group of integers. 4

   (b) $(G, 0)$ and $(G', *)$ are two groups. $f : G \rightarrow G'$ is an onto homomorphism. Prove that if H is a normal subgroup of $(G, 0)$, then $f(H)$ is also a normal subgroup of $(G', *)$. 4

Section- III
(Boolean Algebra)

Answer any *one* question from the following: 7×1 = 7

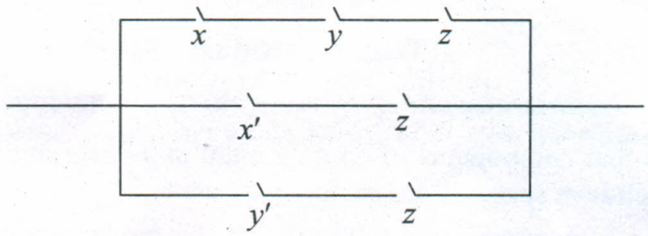
5. (a) Let $f(x, y, z)$ be a Boolean function. f is such that it assumes the value 1 if only one of the variables takes the value 0. Construct a truth table of f and hence write f in DNF and draw a switching circuit corresponding to the DNF. 5

(b) Prove that for any two elements a, b of a Boolean algebra $(B, +, \cdot, ')$, $a + b = 0 \Rightarrow a = 0$ and $b = 0$. 2

6. (a) Does there exist a Boolean algebra with only three elements? Justify your answer. 2

(b) In a Boolean algebra $(B, +, \cdot, ')$, prove that $a + b = a + c$ and $a.b = a.c$ imply $b = c$. 2

(c) Find the Boolean function which represents the circuit and simplify the function, if possible. 3



Group- B

(Differential Equations-III)

Answer any *one* question from the following: $15 \times 1 = 15$

7. (a) Solve the equation 5

$$\frac{d^2y}{dx^2} + (x-1)^2\frac{dy}{dx} - 4(x-1)y = 0$$

in series about the ordinary point $x = 1$

(b) Find the Laplace transform of 5

$$g(t) = \begin{cases} 0, & 0 < t < 5 \\ t-3, & t > 5. \end{cases}$$

(c) Solve the initial value problem using Laplace transform: $y' - 2y = e^{5t}, y(0) = 3$. 5

8. (a) Obtain a power series solution of the initial value problem $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$, $y(0) = 0$, $y'(0) = 1$ in powers of x. 5

(b) Using convolution theorem, find $\mathcal{L}^{-1}(L(t))$ where $L(t)=\dfrac{1}{s^2+5s+6}$. 5

(c) Apply Laplace transform to solve: 5

$$y''+4y'+5y=e^t$$
$$y(0)=1$$
$$y'(0)=2.$$

Group- C

(Tensor Calculus)

Answer any *one* question from the following: $10\times1=10$

9. (a) Prove that components of contravariant and covariant vectors in Euclidean space of dimension n are same. 2

(b) Define Riemannian space. Show that the fundamental metric tensor g_{ij} is a symmetric (0, 2) type tensor. 2+3

(c) Prove that $\begin{Bmatrix} i \\ i \;\; j \end{Bmatrix}=\dfrac{\partial}{\partial x^j}\left(\log\sqrt{g}\right)$, where $g=\left|g_{ij}\right|$. 3

10. (a) Write the cosine expression of angle between two non-null vectors A^i and B^i in Riemannian space. When they are orthogonal? 1+1

(b) If A^i and B^i are two non-null vectors such $g_{ij}\,U^i\,U^j=g_{ij}\,V^i\,V^j$ where $U^i=A^i+B^i$ and $V^i=A^i-B^i$. Show that A^i and B^i are orthogonal. 2

(c) Prove that Christoffel symbols are not tensor. 3

(d) Define covariant differentiation of a vector. Give geometric interpretation of it. 1+2